

Implication of Dark Energy Parametrizations on the Determination of the Curvature of the Universe

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Abstract

We investigate how the nature of dark energy affects the determination of the curvature of the universe from recent observations. For this purpose, we consider the constraints on the matter and dark energy density using observations of type Ia supernovae, baryon acoustic oscillation peak and cosmic microwave background with several types of dark energy equation of state. Although it is usually said that the combination of current observations favors a flat universe, we found that a relatively large parameter space allows the universe to be open for a particular model of dark energy. We also discuss what kind of dark energy model or prior allow a non-flat universe.

1 Introduction

The nature of dark energy is one of the challenging mysteries in today's science. Although cosmological observations such as cosmic microwave background (CMB) is becoming very precise and a lot of efforts have been made to understand it, we still do not know what the dark energy is. One can try to investigate it phenomenologically using the dark energy equation of state w_X as a parameter which describes the nature of dark energy and constraining it using various observations. In the early stage of the study of dark energy, the equation of state for dark energy has been assumed to be constant in time. However, many models proposed so far have time-varying equation of state, thus most of recent phenomenological works for dark energy accommodate its time dependence in some way.

In such works, constraints on the time dependence of the equation of state for dark energy is often obtained assuming a flat universe. The assumption of a flat universe seems to be reasonable since it is usually said that the constraint on the curvature of the universe from a combination of various data sets suggests that the universe is flat. But notice that, when one discusses the constraint on the curvature of the universe, the cosmological constant is usually assumed as dark energy. It is not so obvious that a flat universe is preferred when one assumes different models for dark energy. Similarly, when one considers the constraint on the nature of dark energy such as the equation of state, a flat universe is assumed in many literatures, but it is also not so trivial how the curvature of the universe affects the constraint on the nature of dark energy.

Thus it is important to study the curvature of the universe without assuming a cosmological constant for dark energy or the dark energy equation of state without assuming a flat universe. In fact, some works along this line have been done in Refs. [1, 2, 3] (for other recent considerations on degeneracies between the equation of state for dark energy and the curvature of the universe, we refer to Refs. [4, 5, 6, 7, 8]). In Ref. [1], it was shown that there is a degeneracy between the curvature of the universe and the constant equation of state for dark energy in the CMB angular power spectrum. Furthermore, in Ref. [2, 3], the constraint on the curvature of the universe was investigated using a particular parameterization of dark energy which accommodates the time dependence in some way. It was shown that, even if we assume a time-varying equation of state for dark energy, one can obtain an almost flat universe from the combination of various observational data sets. However, it should be noticed that the robustness of the flatness was shown only for some particular parametrizations for dark energy.

In this paper, we extend the analysis done in Ref. [3] in order to elucidate the role of the dark energy prior on the measurement of the curvature of the universe. Especially, we will show that there exists a type of dark energy time-dependence which allows significantly larger region of an open universe than that allowed by the dark energy model adopted in Ref. [3] (and in many other works). It should be emphasized that this holds even if we combine recent observations which measure cosmological distances to various redshifts. We consider the observations of type Ia supernovae (SNeIa), CMB and baryon acoustic oscillation (BAO) peak. For the CMB constraint, we make use of the shift parameter

which well captures the whole shift of the CMB angular power spectrum.

The structure of this paper is as follows. In the next section, after we discuss the parametrization of the dark energy equation of state, we briefly summarize the analysis method adopted in this paper. Then, in section 3, we study the constraints on the curvature of the universe. The final section is devoted to the summary of this paper.

2 Analysis method

In this section, first we summarize the parameterization for the dark energy equation of state which we are going to use in our analysis.

We consider the following parametrization for the dark energy equation of state,

$$w_X(z) = \begin{cases} w_0 + \frac{w_1 - w_0}{z_*} z & (\text{for } z \leq z_*) \\ w_1 & (\text{for } z \geq z_*), \end{cases} \quad (2.1)$$

where we interpolate the value of w_X linearly from the present epoch back to the transition redshift z_* . w_X becomes w_0 at $z = 0$ and w_1 at $z \geq z_*$. The energy density of dark energy in this parametrization is calculated as

$$\rho_X(z)/\rho_X(z=0) = \begin{cases} \exp[3\alpha z](1+z)^{3(1+w_0-\alpha)} & (\text{for } z \leq z_*) \\ \exp[3\alpha z_*](1+z_*)^{3(1+w_0-\alpha)} \left(\frac{1+z}{1+z_*}\right)^{3(1+w_1)} & (\text{for } z \geq z_*), \end{cases} \quad (2.2)$$

where

$$\alpha \equiv \frac{w_1 - w_0}{z_*}. \quad (2.3)$$

This parametrization is essentially the same as the one which has a linear dependence on the redshift z such as $w_X = w_0 + \alpha z$. In fact, this form is adopted in many literatures with a cutoff at some redshift to avoid large value of w_X at $z \sim 1000$ which is relevant to the CMB constraint [9, 10, 11]. In this paper, we consider several value of z_* and show how the values of z_* affect the constraint on the curvature.

Now we briefly discuss the analysis method for constraining the model and cosmological parameters. For the purpose of the present paper, we make use of observational data of SNeIa, CMB and baryon acoustic oscillation ^{#1}. To compare a model with observational

^{#1}To obtain the constraint on the nature of dark energy, some authors also make use of the angular size of compact radio sources and/or X-ray gas mass fraction of galaxy cluster Ref.[12, 13, 14, 15]. However we do not consider them here.

data, first we need to know the evolution of the Hubble parameter which can be written as

$$H^2(z) = H_0^2 \left[\Omega_r(1+z)^4 + \Omega_m(1+z)^3 + \Omega_k(1+z)^2 + \Omega_X \exp \left(3 \int_0^z (1+w_X(\bar{z})) \frac{d\bar{z}}{1+\bar{z}} \right) \right] \quad (2.4)$$

where H_0 is the Hubble parameter at the present time, Ω_i is the energy density at the present epoch normalized by the critical energy density and the subscripts r, m, k and X represent radiation, matter, the curvature and dark energy, respectively.

For the SNeIa data, we fit the distance modulus to observational data from the gold data set given in Ref. [16] and the Supernova Legacy survey [17]. The distance modulus is defined as

$$M - m = 5 \log d_L + 25. \quad (2.5)$$

Here d_L is the luminosity distance in units of Mpc which is written as

$$d_L = \frac{1+z}{\sqrt{|\Omega_k|}} \mathcal{S} \left(\sqrt{|\Omega_k|} \int_0^z \frac{dz'}{H(z')/H_0} \right) \quad (2.6)$$

where \mathcal{S} is defined as $\mathcal{S}(x) = \sin(x)$ for a closed universe, $\mathcal{S}(x) = \sinh(x)$ for an open universe and $\mathcal{S}(x) = x$ with the factor $\sqrt{|\Omega_k|}$ being removed for a flat universe.

We also use the observation of baryon oscillation acoustic peak which has been detected in the SDSS luminous red galaxy sample and recently used to constrain the properties of dark energy by many authors. The quantity we use to constrain the cosmological parameters is the parameter A which is defined as

$$A = \frac{\sqrt{\Omega_m}}{(H(z_1)/H_0)^{1/3}} \left[\frac{1}{z_1 \sqrt{|\Omega_k|}} \mathcal{S} \left(\sqrt{|\Omega_k|} \int_0^{z_1} \frac{dz'}{H(z')/H_0} \right) \right]^{2/3} \quad (2.7)$$

where $z_1 = 0.35$ and A is measured as $A = 0.469 \pm 0.017$ [18].

To obtain the constraint from CMB observations, we make use of the so-called shift parameter which measures the whole shift of the CMB angular power spectrum. The shift parameter is usually written as R and defined as

$$R = \frac{\Omega_m}{\sqrt{|\Omega_k|}} \mathcal{S} \left(\sqrt{|\Omega_k|} \int_0^{z_2} \frac{dz'}{H(z')/H_0} \right) \quad (2.8)$$

where $z_2 = 1089$, the redshift of the epoch of the recombination. From the three-year WMAP result [19, 20, 21, 22], the shift parameter is constrained to be $R = 1.70 \pm 0.03$ [23]. Since we only consider the shift parameter which is determined only by the background evolution for the constraint from CMB, we do not need to include the effect of the fluctuation of dark energy. In this paper, by using shift parameter, we can confine ourselves to considering the effects of the modification of the background evolution alone.

Here, some comments on this are in order. The reason why we only consider the background evolution is twofold. The first one is that the constraint on dark energy almost comes from the fit to the position of the acoustic peaks although the fluctuation of dark energy component can affect the low multipole region of the CMB angular power spectrum. As mentioned above, the shift parameter is the measure of the position of the acoustic peaks. Thus the use of the shift parameter is a robust way to obtain the constraint from CMB. Another reason is that we need to know more about the nature of dark energy to include the effects of the fluctuation of it such as its speed of sound. Furthermore, in some models, the Friedmann equation itself is modified to explain the present cosmic acceleration such as the DGP model [24]. Even in such models, one can define the effective equation of state for a “fictitious” dark energy component such that the background evolution is the same as the original models. However, as for the perturbation, we need to take the modification into account properly, which cannot be done by just assuming a “fictitious” dark energy in most cases. By considering the observable which is only affected by the modification of background we can make our analysis general, accomodating both ordinary dark energy and modified gravity.

Using the observations mentioned above, we perform χ^2 analysis to obtain the constraints on energy densities of matter and dark energy with its equation of state which varies as Eq. (2.1). In the next section, we discuss the constraints on them in detail.

3 Constraints on the curvature of the universe

In this section, we discuss the constraint on the curvature of the universe using observations mentioned in the previous section. We assume the parametrization introduced in the previous section for the equation of state of dark energy.

Now we present the constraint on the Ω_m vs. Ω_X plane. In Fig. 1, contours of 1σ and 2σ constraints from all data combined (i.e., SNeIa, CMB and BAO) are shown. In the figure, we fixed the value of the transition redshift z_* as $z_* = 0.1, 0.5, 0.9, 1.3, 1.7$ and 2.0 . To obtain the constraint, we marginalized over the values of w_0 and w_1 . We assumed the prior on w_0 and w_1 as $-5 < w_0 < 0$ and $-5 < w_1 < 0$. We note that χ^2 minimum is not given by the lower bound of these priors for any (Ω_m, Ω_X) . As seen from the figure, for several values of the transition redshift z_* , the allowed region extends to the parameter space where the curvature of the universe is negative, namely the universe is open. This seems contrary to the result of Ref. [3] where we have shown the same combined data set (to be exact, since we used the WMAP 1st year value for the shift parameter in Ref. [3], its error was two times as large as the three-year value we adopt in this paper) favors the region closely around a flat universe (see Fig. 5d in Ref. [3]) for the dark energy model

$$w_X = \tilde{w}_0 + (1 - a)\tilde{w}_1, \quad (3.9)$$

upon marginalizing over \tilde{w}_0 and \tilde{w}_1 . In the following, we will give explanations to our result paying particular attention to the difference between the models examined in Ref. [3] and Eq. (2.1).

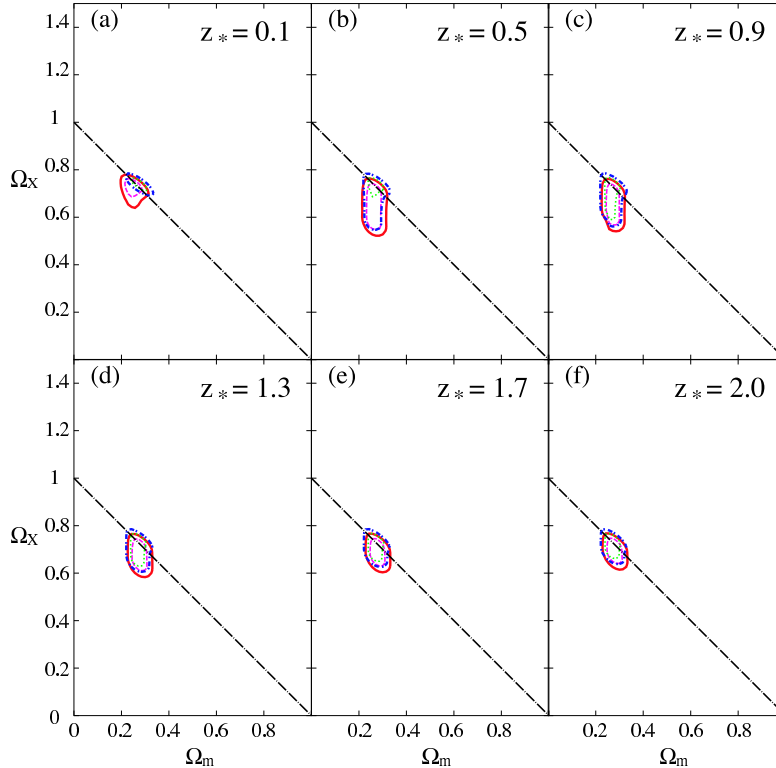


Figure 1: Constraints from all combined data set on the Ω_m vs. Ω_X plane for the parametrization of Eq. (2.1). We marginalized over w_0 and w_1 . 1σ and 2σ contours using gold (SNLS) data set for SNeIa are shown with magenta dashed (green dotted) and red solid (blue dot-dashed) lines respectively. We fixed the value of the transition redshift as $z_* = 0.1$ (a), 0.5 (b), 0.9 (c), 1.3 (d), 1.5 (e) and 2.0 (f).

Before interpreting the results in Fig. 1, we should note that although the model defined by Eq. (2.1) appears to have three parameters w_0, w_1 and z_* we regard it as a family of two-parameter (w_0 and w_1) models labeled by z_* . Namely, we consider several two-parameter models each with different value of z_* . Such distinction among dark energy parameters may be artificial, but it is sufficient for our goal to show the existence of some models which fit the data set with a non-flat universe. Since the minimum values of χ^2 for each model with fixed z_* are smaller than the χ^2 minimum for the case of cosmological constant, it is at least sensible to consider such model parametrization. For example, the total χ^2 minimum for cosmological constant is 177.99 when gold data is used for SNeIa while the model with $z_* = 0.5$ gives χ^2 minimum as 173.50.

Now we look into Fig. 1 more closely. For the cases with a small transition redshift as $z_* \sim 0.1$ and a large transition redshift like $z_* \sim 2.0$, the region around a flat universe is favored (Fig. 1 a, f). Since the equation of state approaches to the case with a constant equation of state $w_X = w_1$ as $z_* \rightarrow 0$, the constraint for small z_* is expected to be

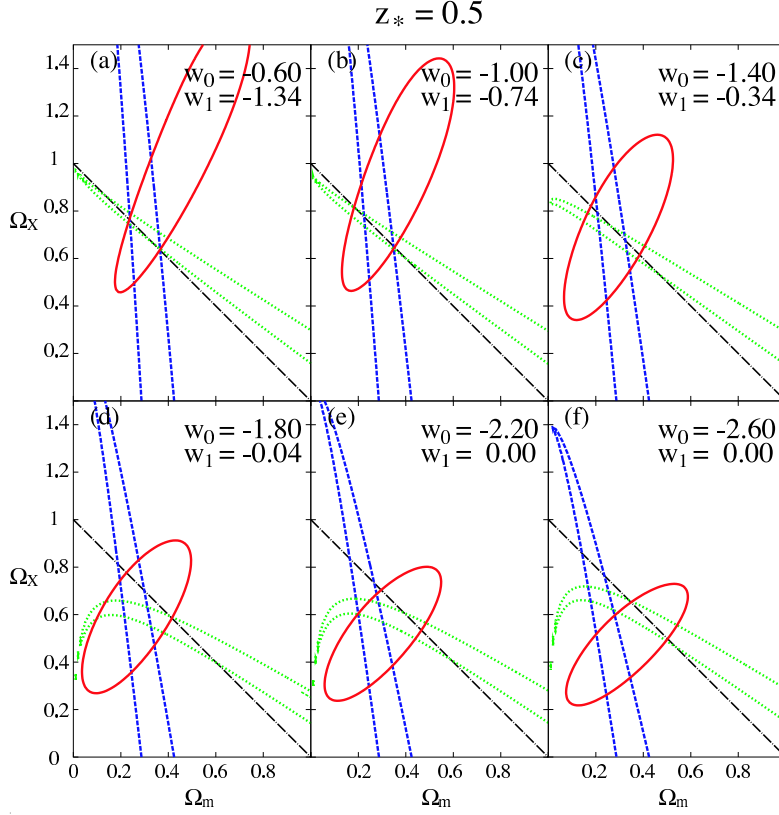


Figure 2: 2σ constraints on the Ω_m vs. Ω_X plane from SNeIa (red solid line), BAO (blue dashed line), CMB shift parameter (green dotted line) are shown for the models with the transition redshift $z_* = 0.5$. w_0 is fixed to be -0.6 (a), -1.0 (b), -1.4 (c), -1.8 (d), -2.2 (e) and -2.6 (f). w_1 is taken to the value which gives minimum combined χ^2 on each panel, that is, $w_1 = -1.34$ (a), -0.74 (b), -0.34 (c), -0.04 (d), 0.00 (e) and 0.00 (f).

similar to the one obtained for a constant w_X . This case has been investigated in Ref. [3] and found to favor the region closely around a flat universe, consistent with the result of $z_* = 0.1$ here. Meanwhile, the model with $z_* = 2.0$ has a similar w_X evolution to the case of Eq. (3.9). This case has also been investigated in Ref. [3] and again shown to favor the region around a flat universe. Thus, the result for $z_* = 2.0$ in this paper is considered to be reasonable.

On the other hand, for the case with an intermediate transition redshift $0.5 \lesssim z_* \lesssim 1.0$, the situation is different. We can see that the allowed region include a relatively large region of an open universe (Fig. 1 b, c). Since we marginalize over w_0 and w_1 , it is not easy to understand how this happens. Thus, in Figs. 2 and 3, we also show the constraints from each observation separately fixing these parameters respectively with $z_* = 0.5$ and 2.0 . Here, gold data set is used for SNeIa data. In these figures, we show the constraints on Ω_m and Ω_X for the case with w_0 fixed to be -0.6 (a), -1.0 (b), -1.4 (c), -1.8 (d),

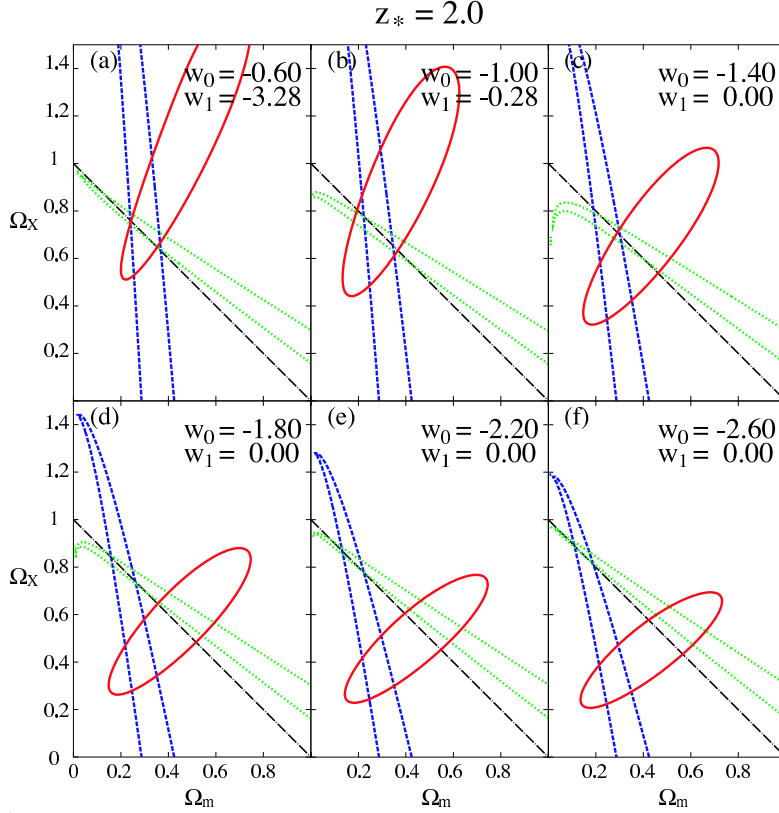


Figure 3: The same as Fig. 2 except that $z_* = 2.0$. w_1 is taken to be -3.28 (a), -0.28 (b) and 0.0 (c-f).

-2.2 (e) and -2.6 (f). For each w_0 , we choose w_1 to give the minimum of total χ^2 . Then in Fig. 2 ($z_* = 0.5$), $w_1 = -1.34$ (a), -0.74 (b), -0.34 (c), -0.04 (d), 0.00 (e) and 0.00 (f). In Fig. 3 ($z_* = 2.0$), $w_1 = -3.28$ (a), -0.28 (b) and 0.0 (c-f).

A quick inspection of those figures reveals the difference of $z_* = 0.5$ and 2.0 in Fig. 1 (b and f). In Fig. 2, constraints from three observations cross at region around a flat universe with $\Omega_m \sim 0.3$ when $w_0 \gtrsim -1.5$ and $w_1 \lesssim -0.3$ (a-c). In addition, the observations allow an open universe with $w_0 = -1.8$ and $w_1 \sim 0$ (d), which corresponds to the elongated feature of the contours in Fig. 1 (b). By contrast, in Fig. 3, the observations only allow the region around a flat universe (a, b) and we do not find dark energy parameters such as to allow an open universe, as is shown in Fig. 1 (f).

Since the contours of SNeIa and BAO have relatively similar degeneracy pattern with respect to the change of w_0 and do not depend much on the value of w_1 (these observations measure the distances to lower z so they are controlled by w_0 rather than w_1), we should look at CMB constraints to understand the difference between Figs. 2 and 3. This is most noticeable on comparing panel (d) of the two figures. In Fig. 2, since the model has the transition redshift $z_* = 0.5$ which is close to the present epoch, the value of w_1

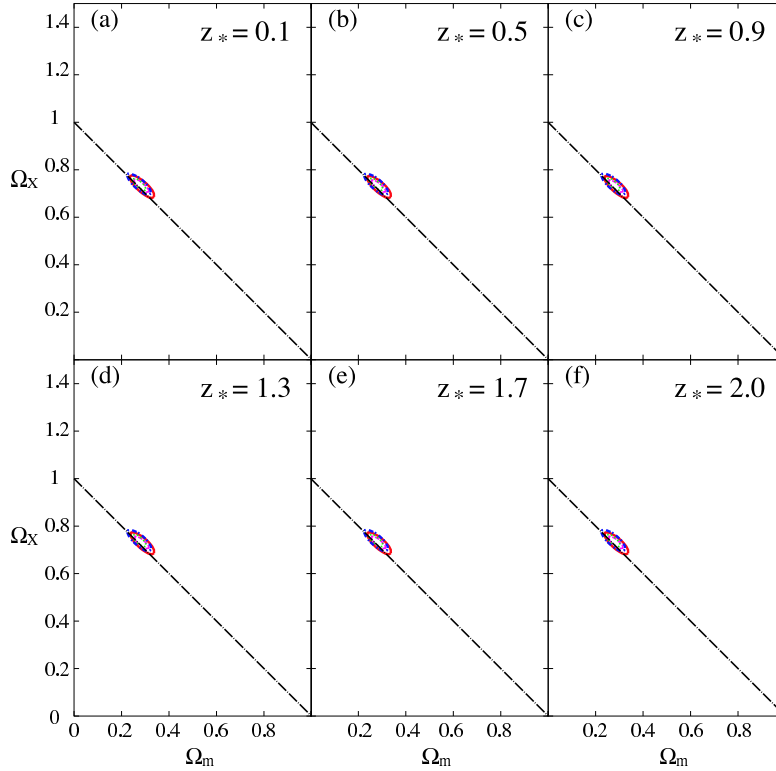


Figure 4: The same as Fig. 1 except that we assumed the prior $w_X \leq -1$.

mainly matters for the distance to the last scattering surface. Then the dark energy with $w_1 \sim 0$ leads to shorter comoving distance to the last scattering surface. This cancels the geometrical effect of an open universe and the angular diameter distance to the last scattering surface can have the observed value. Such cancellation, however, does not seem to take place in Fig. 3 (d) where the transition redshift is as high as $z_* = 2.0$ so the value of w_0 affects the acceleration. With $w_0 = -1.8$, since the universe experiences significant amount of acceleration at late times, the observed angular diameter distance is not produced in an open universe. From these considerations, we can conclude that the value of the transition redshift z_* plays important role in measuring the curvature of the universe.

From above discussion, we have noticed that, when an open universe is allowed, the typical parameter range of the equation of state with the parametrization of Eq. (2.1) is that $w_0 \leq -1$, $w_1 \sim 0$ and $0.5 \lesssim z_* \lesssim 1.0$. Since we need a large value for w_1 such as $w_1 \sim 0$ for an open universe, it can be expected that we would have a flat universe if we assume the prior on the equation of state as $w_X \leq -1$ (for the parameterization adopted here, this also means that $w_1 \leq -1$), i.e., so-called the phantom dark energy. To show this explicitly, we also show the constraint on the Ω_m vs. Ω_X plane with the prior $w_X \leq -1$ for several values of z_* in Fig. 4. We can see that the region around a flat universe only

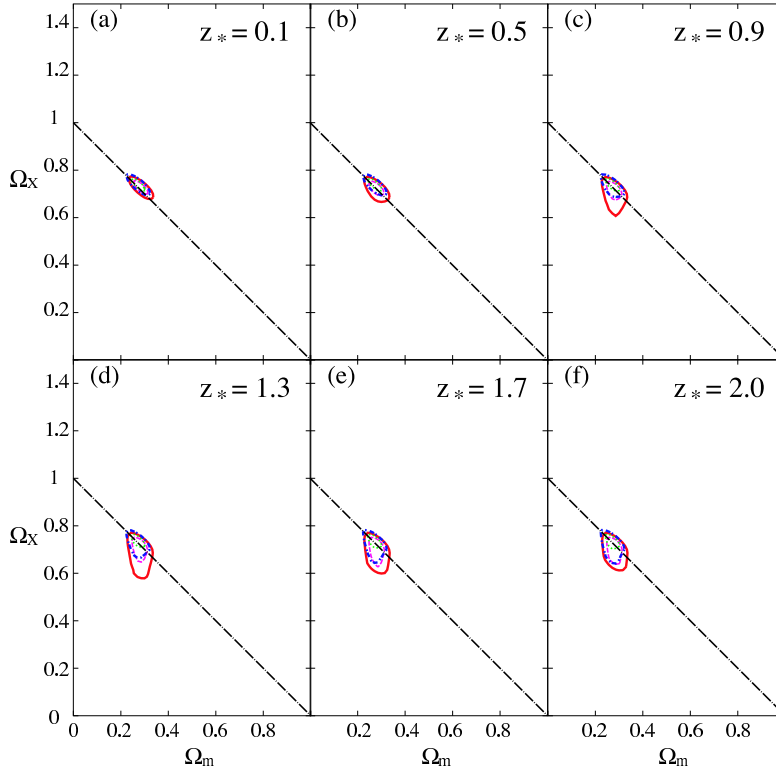


Figure 5: The same as Fig. 1 except that we assumed the prior $w_X \geq -1$.

is favored in this case. This is because w_1 can not be as large as 0 in phantom case by definition. For reference, the constraint for the case with $w_X \geq -1$ is also shown in Fig. 5. In this case, although w_0 cannot be less than -1 , the values of w_1 can be large, which realizes the situation where an open universe is favored to some extent. Thus the allowed region slightly extends to the parameter space of an open universe for some values of z_* .

Here we make some comments on the difference between the parametrization adopted in this note and that of Eq. (3.9) which was investigated in Ref. [3]. Although both parametrizations accommodate the time dependence of a dark energy equation of state, they are different in terms of how rapidly w_X changes. The parametrization we adopt here can rapidly change the value of w_X around the redshift where observations of SNeIa and BAO mostly probe for suitable choice of the transition redshift. On the other hand, for the models with Eq. (3.9), the equation of state changes its value more gradually compared to the parametrization used here. Thus such cases is more or less similar to the case with a constant equation of state, which is the reason why we got an almost flat universe even if we allow w_X to vary in time. We have also made the analysis for the models where the equation of state changes with a step function at some transition redshift as $w_X = w_0\theta(z_* - z) + w_1\theta(z - z_*)$. We obtained almost the same tendency for the constraint as the parametrization of Eq. (2.1). This means that models of dark energy which have

a rapidly changing equation of state around the redshift range $0.5 \lesssim z_* \lesssim 1$, large region of an open universe can be allowed. Thus the assumption of a flat universe may not be a valid assumption when one tries to constrain dark energy model parameters for some particular models.

4 Conclusion and discussion

We discussed how the nature of dark energy affects the determination of the curvature of the universe. In fact, it has been explicitly shown that, when one adopts a dark energy model with a constant equation of state or the parametrization with $w_X = \tilde{w}_0 + (1 - a)\tilde{w}_1$, current cosmological observations constrain the curvature of the universe to be almost flat [3]. In this note, we considered this issue using another parametrization which has the form of Eq. (2.1). We found that a flat universe is not always favored for some particular cases. As shown in Fig. 1, when the transition redshift is in the range of $0.5 \lesssim z_* \lesssim 1$, the allowed region significantly extends to the parameter space of an open universe.

We have also investigated what kind of dark energy can allow an open universe by considering the constraint in detail. We have found that an open universe tends to be most favored when rapid transition such that $w_0 \leq -1$, $w_1 \sim 0$ and $0.5 \lesssim z_* \lesssim 1$ occurs. However, the period of $w_X < -1$ is not necessary for an open universe to be allowed and actually we have shown the allowed region extends to the space of an open universe to some extent although it does not cover large region with the prior of conventional dark energy: $w_X(z) \geq -1$. Finally, when the phantom dark energy $w_X(z) \leq -1$ is assumed, the allowed region in the Ω_m vs. Ω_X plane lies around the parameter space of a flat universe. Therefore, the dark energy has to behave like matter at early times, $w_1 \sim 0$, in order for an open universe to be allowed.

The result presented in this note may have much implication for the strategies to constrain models of dark energy. Although a flat universe is usually assumed when one studies the nature of dark energy, we have noticed that, for some models of dark energy, an open universe is more favored than a flat universe. Thus a flat universe may not be a good assumption to constrain some particular models of dark energy. On the other hand, the above statement also means that one should be careful to assume a model of dark energy when one tries to determine the curvature of the universe since the assumption may affect the constraint on it.

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